Applied Mathematics and Mechanics
(English Edition, Vol 25, No 7, Jul 2004)

Published by Shanghai University,
Shanghai, China

Article ID: 0253-4827(2004)07-0730-11

INVESTIGATION OF THE BEHAVIOR OF A GRIFFITH CRACK AT THE INTERFACE BETWEEN TWO DISSIMILAR ORTHOTROPIC ELASTIC HALF-PLANES FOR THE OPENING CRACK MODE *

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Abstract: *The behaviors of an interface crack between dissimilar orthotropic elastic half-planes subjected to uniform tension was reworked by use of the Schmidt method. By use of the Fourier transform, the problem can be solved with the help of two pairs of dual integral equations, of which the unknown variables are the jumps of the displacements across the crack surfaces. Numerical examples are provided for the stress intensity factors of the cracks. Contrary to the previous solution of the interface crack, it is found that the stress singularity of the present interface crack solution is of the same nature as that for the ordinary crack in homogeneous materials. When the materials from the two half planes are the same, an exact solution can be otained.*

Key words: interfacial crack; Schmidt method; dual integral equation; orthotropic material

Chinese Library Classification: O346.33 **Document code**: A

2000 Mathematics Subject Classification: 74R10

Introduction

In recent years, composite materials and adhesive or bonded joints are being used in wide range of engineering field. The fracture of composites and bonded dissimilar materials is induced mainly from the interfacial region because the angular corner of bonded materials induces singular stress and crack initiation at the interface. Particularly flaws or cracks lying along the interface reduce the strength of the structure significantly. Hence, problem of interface cracks in dissimilar materials is very important from the view point of interface strength and stress analysis of interface cracks have been treated in Refs. [1 ~ 9]. For the interface crack problem, it is well known that the stress oscillatory singularity and overlapping of the crack surfaces appear near the interface

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* **Received date**: 2003-04-15; **Revised date**: 2004-03-10

Foundation item: the National Natural Science Foundation of China (10172030,50232030); the Natural Science Foundation of Heilongjiang Province of China (A0301)

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crack tip and these are quite different from ordinary cracks in homogeneous materials. Therefore, in comparison with the ordinary crack problems, it is difficult of analyze accurately the interface crack problems. Up to the present, we do not think that this problem has been completely solved. In Refs.[9 ~ 11], the stress oscillatory singularity and overlapping of the crack surfaces do not appear near the interface crack tip for the opening interface crack mode. Some of the more significant results, particularly that concerning the discussion of the conditions leading to non-oscillating crack tip stress fields were given in Refs.[12 ~ 19]. However, the interface crack model had been changed, i.e., the tips of the crack were assumed to be closed. In Achenbach's work[20], the interface crack problem was also studied. Non-oscillating crack tip stress fields were obtained in Ref.[20]. However, it was assumed that there was an adhesive zone at the crack tips.

Mathematically, the solutions in Refs. [1 ~ 3] are exact forms in spite of the incomprehensibility in fracture mechanics. This is probably caused by the unreasonable crack model (It is assumed that two crack surfaces lie on the same line and there is an opening displacement on the crack surfaces simultaneously). However, from an engineering viewpoint, it is more desirable to seek a solution which is physically acceptable[11]. In the present paper, the same problem which was treated by Zhang[9] is reworked by use of a somewhat different approach, named the Schmidt method[21 ~ 22]. It is a simple and convenient method for solving this problem. As in many previous studies[8], in this study too, the problem is solved under the assumption that the effect of the crack surface interference very near the crack tips is negligible and there is a sufficiently large component of mode I loading so that the crack essentially remains open. The Fourier transform technique is applied and a mixed boundary value problem is reduced to two pairs of dual integral equations in which the unknown variables are the jumps of the displacements across the crack surfaces. To solve the dual integral equations, the jumps of the displacements across the crack surfaces are expanded in a series of Jacobi polynomials. This process is quite different from those adopted in Refs.[1 ~ 20] as mentioned above. Numerical solutions are obtained for the stress intensity factors. Contrary to the previous solution of the interface crack, it is found that the stress singularity of the present interface crack solution is of the same nature as that for the ordinary crack in homogeneous materials. When the materials from the two half planes are the same, an exact solution can be obtained.

1 Formulation of the Problem

It is assumed that there is an interface crack of length $2l$ along the x-axis between two dissimilar orthotropic elastic half-planes $-\infty < x < \infty, 0 \leqslant y < \infty$ and $-\infty < x < \infty$, $-\infty < y \leqslant 0$ as shown in Fig. 1. The elastic constants involved in this problem are denoted by $E_i^{(j)}, \mu_{ik}^{(j)}$ and $\nu_{ik}^{(j)} (i, k = 1,2,3)$, where $j = 1,2$ corresponds to the half-planes $y \geqslant 0$ and $y \leqslant 0$. Dimensionless components of the displacement in x-, y-directions are assumed to be $u^{(j)}, v^{(j)}$, respectively, where $u^{(j)} = u^{(j)}(x, y)$ and $v^{(j)} = v^{(j)}(x, y)$. The nonzero stress components $\sigma_{xy}^{(i)}$ and

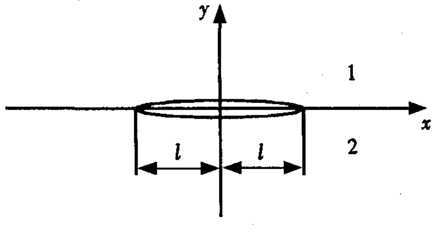

Fig.1 Geometry and coordinate system for the interface crack

$\sigma_{yy}^{(i)}$ are given by

$$\frac{\sigma_{yy}^{(j)}}{\mu_{12}^{(j)}} = c_{12}^{(j)}\frac{\partial u^{(j)}}{\partial x} + c_{22}^{(j)}\frac{\partial v^{(j)}}{\partial y} \quad (j = 1,2), \tag{1}$$

$$\frac{\sigma_{xy}^{(j)}}{\mu_{12}^{(j)}} = \frac{\partial u^{(j)}}{\partial y} + \frac{\partial v^{(j)}}{\partial x} \quad (j = 1,2). \tag{2}$$

The non-dimensional parameters $c_{ik}^{(j)}(i,k = 1,2,3, j = 1,2)$ involved in the above equations are related to the elastic constants by the relations:

$$\begin{cases} c_{11}^{(j)} = E_1^{(j)}/[\mu_{12}^{(j)}(1 - \nu_{12}^{(j)^2}E_2^{(j)}/E_1^{(j)})], \\ c_{22}^{(j)} = E_2^{(j)}/[\mu_{12}^{(j)}(1 - \nu_{12}^{(j)^2}E_2^{(j)}/E_1^{(j)})] = c_{11}^{(j)}E_2^{(j)}/E_1^{(j)}, \\ c_{12}^{(j)} = \nu_{12}^{(j)}E_2^{(j)}/[\mu_{12}^{(j)}(1 - \nu_{12}^{(j)^2}E_2^{(j)}/E_1^{(j)})] = \nu_{12}^{(j)}c_{22}^{(j)} = \nu_{21}^{(j)}c_{11}^{(j)} \quad (j = 1,2) \end{cases} \tag{3}$$

for generalized plane stress, and by

$$\begin{cases} c_{11}^{(j)} = E_1^{(j)}(1 - \nu_{23}^{(j)}\nu_{32}^{(j)}/(\Delta^{(j)}\mu_{12}^{(j)}), \\ c_{22}^{(j)} = E_2^{(j)}(1 - \nu_{13}^{(j)}\nu_{31}^{(j)}/(\Delta^{(j)}\mu_{12}^{(j)}), \\ c_{12}^{(j)} = E_1^{(j)}(\nu_{21}^{(j)} + \nu_{13}^{(j)}\nu_{32}^{(j)}E_2^{(j)}/E_1^{(j)})/(\Delta^{(j)}\mu_{12}^{(j)}) = \\ \qquad E_2^{(j)}(\nu_{12}^{(j)} + \nu_{23}^{(j)}\nu_{31}^{(j)}E_1^{(j)}/E_2^{(j)})/(\Delta^{(j)}\mu_{12}^{(j)}), \\ \Delta^{(j)} = 1 - \nu_{12}^{(j)}\nu_{21}^{(j)} - \nu_{23}^{(j)}\nu_{32}^{(j)} - \nu_{31}^{(j)}\nu_{13}^{(j)} - \\ \qquad \nu_{12}^{(j)}\nu_{23}^{(j)}\nu_{31}^{(j)} - \nu_{13}^{(j)}\nu_{21}^{(j)}\nu_{32}^{(j)} \quad (j = 1,2) \end{cases} \tag{4}$$

for plane strain. The constants $E_i^{(j)}$ and $\nu_{ik}^{(j)}(i,k = 1,2,3)$ satisfy Maxwell's relation:

$$\nu_{ik}^{(j)}/E_i^{(j)} = \nu_{ki}^{(j)}/E_k^{(j)}. \tag{5}$$

In this paper, we just consider the generalized plane stress problem.

The equations of equilibrium of the orthotropic materials, in the absence of body forces, may be expressed as follows:

$$c_{11}^{(j)}\frac{\partial^2 u^{(j)}}{\partial x^2} + \frac{\partial^2 u^{(j)}}{\partial y^2} + (1 + c_{12}^{(j)})\frac{\partial^2 v^{(j)}}{\partial x \partial y} = 0 \quad (j = 1,2), \tag{6}$$

$$c_{22}^{(j)}\frac{\partial^2 v^{(j)}}{\partial y^2} + \frac{\partial^2 v^{(j)}}{\partial x^2} + (1 + c_{12}^{(j)})\frac{\partial^2 u^{(j)}}{\partial x \partial y} = 0 \quad (j = 1,2). \tag{7}$$

These equations are to be solved subject to the boundary conditions:

$$\sigma_{yy}^{(1)} = \sigma_{yy}^{(2)} = -\sigma_0;\ \sigma_{xy}^{(1)} = \sigma_{xy}^{(2)} = 0,\ |x| \leqslant l,\ y = 0, \tag{8}$$

$$u^{(1)} = u^{(2)};\ v^{(1)} = v^{(2)};\ \sigma_{yy}^{(1)} = \sigma_{yy}^{(2)};\ \sigma_{xy}^{(1)} = \sigma_{xy}^{(2)},\ |x| \leqslant l,\ y = 0, \tag{9}$$

$$u^{(j)} = v^{(j)} = 0;\ \sigma_{yy}^{(j)} = \sigma_{xy}^{(j)} = 0 \text{ for } \sqrt{x^2 + y^2} \to \infty \quad (j = 1,2). \tag{10}$$

2 Solutions

Because of the symmetry, it suffices to consider the problem for $x \geqslant 0$, $|y| < \infty$. As discussed in Ref.[9], Eqs.(6) ~ (7) can be solved giving

$$u^{(1)}(x,y) = \frac{2}{\pi}\int_0^\infty [A_1(s)e^{-\gamma_1 sy} + A_2(s)e^{-\gamma_2 sy}]\sin(sx)ds, \tag{11}$$

$$v^{(1)}(x,y) = \frac{2}{\pi}\int_0^\infty [\alpha_1 A_1(s)e^{-\gamma_1 sy} + \alpha_2 A_2(s)e^{-\gamma_2 sy}]\cos(sx)ds, \tag{12}$$

$$u^{(2)}(x,y) = \frac{2}{\pi}\int_0^\infty [B_1(s)e^{\gamma_3 sy} + B_2(s)e^{\gamma_4 sy}]\sin(sx)ds, \tag{13}$$

$$v^{(2)}(x,y) = -\frac{2}{\pi}\int_0^\infty [\alpha_3 B_1(s)e^{\gamma_3 sy} + \alpha_4 B_2(s)e^{\gamma_4 sy}]\cos(sx)\mathrm{d}s, \tag{14}$$

where $\alpha_1 = \dfrac{c_{11}^{(1)} - \gamma_1^2}{(1 + c_{12}^{(1)})\gamma_1}$, $\alpha_2 = \dfrac{c_{11}^{(1)} - \gamma_2^2}{(1 + c_{12}^{(1)})\gamma_2}$, $\alpha_3 = \dfrac{c_{11}^{(2)} - \gamma_3^2}{(1 + c_{12}^{(2)})\gamma_3}$, $\alpha_4 = \dfrac{c_{11}^{(2)} - \gamma_4^2}{(1 + c_{12}^{(2)})\gamma_4}$, $A_j(s)$ and $B_j(s)$ $(j = 1,2)$ are unknown functions to be determined.

Quantities $\gamma_j^2 (j = 1,2)$ are real and positive roots of the equation

$$c_{22}^{(1)}\gamma^4 + [c_{12}^{(1)^2} + 2c_{12}^{(1)} - c_{11}^{(1)}c_{22}^{(1)}]\gamma^2 + c_{11}^{(1)} = 0. \tag{15}$$

Quantities $\gamma_j^2 (j = 3,4)$ are real and positive roots of the equation

$$c_{22}^{(2)}\gamma^4 + [c_{12}^{(2)^2} + 2c_{12}^{(2)} - c_{11}^{(2)}c_{22}^{(2)}]\gamma^2 + c_{11}^{(2)} = 0. \tag{16}$$

Substituting Eqs. (11) ~ (14) into Eqs. (1) ~ (2), it can be obtained

$$\frac{\sigma_{yy}^{(1)}}{\mu_{12}^{(1)}} = \frac{2}{\pi}\int_0^\infty s[A_1(s)(c_{12}^{(1)} - c_{22}^{(1)}\alpha_1\gamma_1)e^{-\gamma_1 sy} + A_2(s)(c_{12}^{(1)} - c_{22}^{(1)}\alpha_2\gamma_2)e^{-\gamma_2 sy}]\cos(sx)\mathrm{d}s, \tag{17}$$

$$\frac{\sigma_{xy}^{(1)}}{\mu_{12}^{(1)}} = -\frac{2}{\pi}\int_0^\infty s[A_1(s)(\alpha_1 + \gamma_1)e^{-\gamma_1 sy} + A_2(s)(\alpha_2 + \gamma_2)e^{-\gamma_2 sy}]\sin(sx)\mathrm{d}s, \tag{18}$$

$$\frac{\sigma_{yy}^{(2)}}{\mu_{12}^{(2)}} = \frac{2}{\pi}\int_0^\infty s[B_1(s)(c_{12}^{(2)} - c_{22}^{(2)}\alpha_3\gamma_3)e^{\gamma_3 sy} + B_2(s)(c_{12}^{(2)} - c_{22}^{(2)}\alpha_4\gamma_4)e^{\gamma_4 sy}]\cos(sx)\mathrm{d}s, \tag{19}$$

$$\frac{\sigma_{xy}^{(2)}}{\mu_{12}^{(2)}} = \frac{2}{\pi}\int_0^\infty s[B_1(s)(\alpha_3 + \gamma_3)e^{\gamma_3 sy} + B_2(s)(\alpha_4 + \gamma_4)e^{\gamma_4 sy}]\sin(sx)\mathrm{d}s. \tag{20}$$

By substitutions and using boundary conditions (8) ~ (9), we obtain

$$\mu_{12}^{(1)}[A_1(s)(c_{12}^{(1)} - c_{22}^{(1)}\alpha_1\gamma_1) + A_2(s)(c_{12}^{(1)} - c_{22}^{(1)}\alpha_2\gamma_2)] = \mu_{12}^{(2)}[B_1(s)(c_{12}^{(2)} - c_{22}^{(2)}\alpha_3\gamma_3) + B_2(s)(c_{12}^{(2)} - c_{22}^{(2)}\alpha_4\gamma_4)], \tag{21}$$

$$\mu_{12}^{(1)}[A_1(s)(\alpha_1 + \gamma_1) + A_2(s)(\alpha_2 + \gamma_2)] = -\mu_{12}^{(2)}[B_1(s)(\alpha_3 + \gamma_3) + B_2(s)(\alpha_4 + \gamma_4)]. \tag{22}$$

So jumps of the displacements across the crack surfaces can be defined as follows:

$$f_1(x) = u^{(1)}(x,0) - u^{(2)}(x,0), \tag{23}$$

$$f_2(x) = v^{(1)}(x,0) - v^{(2)}(x,0), \tag{24}$$

where $f_1(x)$ is an odd function, $f_2(x)$ is an even function. $f_i(x)(i = 1,2)$ is an unknown function of x to be determined by the boundary conditions. However, in the previous works, the unknown function is $\dfrac{\partial f_i(x)}{\partial x}(i = 1,2)$, i.e., the dislocation density function.

Applying the Fourier transforms and Eqs. (11) ~ (14) and (23) ~ (24), it can be obtained

$$\bar{f}_1(s) = A_1(s) + A_2(s) - B_1(s) - B_2(s), \tag{25}$$

$$\bar{f}_2(s) = \alpha_1 A_1(s) + \alpha_2 A_2(s) + \alpha_3 B_1(s) + \alpha_4 B_2(s). \tag{26}$$

A superposed bar indicates the Fourier transform. If $f(x)$ is an even function, the Fourier transform is defined as follows:

$$\bar{f}(s) = \int_0^\infty f(x)\cos(sx)\mathrm{d}x,\ f(x) = \frac{2}{\pi}\int_0^\infty \bar{f}(s)\cos(sx)\mathrm{d}s. \tag{27}$$

If $f(x)$ is an odd function, the Fourier transform is defined as follows:

$$\bar{f}(s) = \int_0^\infty f(x)\sin(sx)\mathrm{d}x, \; f(x) = \frac{2}{\pi}\int_0^\infty \bar{f}(s)\sin(sx)\mathrm{d}s. \tag{28}$$

By solving four Eqs. (21) ~ (22) and (25) ~ (26) with four unknown functions, substituting the solutions into Eqs. (17) ~ (18) and applying the boundary conditions (8) ~ (9), it can be obtained

$$\sigma_{yy}^{(1)}(x,0) = \frac{2\mu_{12}^{(1)}}{\pi}\int_0^\infty s[d_1\bar{f}_1(s) + d_2\bar{f}_2(s)]\cos(sx)\mathrm{d}s = -\sigma_0 \qquad (0 \leqslant x \leqslant l), \tag{29}$$

$$\sigma_{xy}^{(1)}(x,0) = -\frac{2\mu_{12}^{(1)}}{\pi}\int_0^\infty s[d_3\bar{f}_1(s) + d_4\bar{f}_2(s)]\sin(sx)\mathrm{d}s = 0 \qquad (0 \leqslant x \leqslant l), \tag{30}$$

$$\int_0^\infty \bar{f}_1(s)\sin(sx)\mathrm{d}s = 0 \qquad (x > l), \tag{31}$$

$$\int_0^\infty \bar{f}_2(s)\cos(sx)\mathrm{d}s = 0 \qquad (x > l), \tag{32}$$

where d_1, d_2, d_3 and d_4 are constants (See Appendix). Other it can be obtained that $d_1 = 0$, $d_2 = \dfrac{(c_{12}^{(1)}c_{12}^{(1)} - c_{11}^{(1)}c_{22}^{(1)})(q_3q_1 + q_0q_1 - q_3q_2 + q_0q_2)}{4\sqrt{2}c_{11}^{(1)}q_0}$, $d_3 = \dfrac{-c_{12}^{(1)}c_{12}^{(1)} + c_{11}^{(1)}c_{22}^{(1)}}{\sqrt{2c_{22}^{(1)}}(\sqrt{q_3 - q_0} + \sqrt{q_3 + q_0})}$ and $d_4 = 0$ if we take $(E_i^{(1)}, \mu_{ik}^{(1)}, \nu_{ik}^{(1)}) = (E_i^{(2)}, \mu_{ik}^{(2)}, \nu_{ik}^{(2)})$ $(i, k = 1,2,3)$. Where $q_1 = \sqrt{\dfrac{q_3 - q_0}{c_{22}^{(1)}}}$, $q_2 = \sqrt{\dfrac{q_3 + q_0}{c_{22}^{(1)}}}$, $q_3 = c_{11}^{(1)}c_{22}^{(1)} - c_{12}^{(1)}(2 + c_{12}^{(1)})$, $q_0 = \sqrt{[c_{12}^{(1)}(2 + c_{12}^{(1)}) - c_{11}^{(1)}c_{22}^{(1)}]^2 - 4c_{11}^{(1)}c_{22}^{(1)}}$. To determine the unknown functions $\bar{f}_1(s)$ and $\bar{f}_2(s)$, the above two pairs of dual integral equations (29) ~ (32) must be solved.

3 Solutions of the Dual Integral Equations

As mentioned above, this problem is solved under the assumption that the effect of the crack surface interference very near the crack tips is negligible and there is a sufficiently large component of mode I loading so that the crack essentially remains open. This assumption had been used in Erdogan's paper[8]. It can be obtained that the jumps of the displacements across the crack surface are finite, differentiable and continuous functions. Hence, the jumps of the displacements across the crack surface can be expanded by the following series:

$$f_1(x) = \sum_{n=0}^\infty a_n P_{2n+1}^{(1/2,1/2)}\left(\frac{x}{l}\right)\left(1 - \frac{x^2}{l^2}\right)^{1/2}, \text{ for } 0 \leqslant x \leqslant l, \tag{33}$$

$$f_1(x) = 0, \text{ for } x > l, \tag{34}$$

$$f_2(x) = \sum_{n=0}^\infty b_n P_{2n}^{(1/2,1/2)}\left(\frac{x}{l}\right)\left(1 - \frac{x^2}{l^2}\right)^{1/2}, \text{ for } 0 \leqslant x \leqslant l, \tag{35}$$

$$f_2(x) = 0, \text{ for } x > l, \tag{36}$$

where a_n and b_n are unknown coefficients, $P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial[23].

The Fourier transform of Eqs. (33) ~ (36) is[24]

$$\bar{f}_1(s) = \sum_{n=0}^\infty a_n G_n^{(1)}\frac{1}{s}\mathrm{J}_{2n+2}(sl), \; G_n^{(1)} = \sqrt{\pi}(-1)^n\frac{\Gamma(2n + 2 + 1/2)}{(2n + 1)!}, \tag{37}$$

$$\bar{f}_2(s) = \sum_{n=0}^{\infty} b_n G_n^{(2)} \frac{1}{s} \mathrm{J}_{2n+1}(sl), \quad G_n^{(2)} = \sqrt{\pi}(-1)^n \frac{\Gamma(2n+1+1/2)}{(2n)!}, \tag{38}$$

where $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

Substituting Eqs.(37) ~ (38) into Eqs.(29) ~ (32), it can be shown that Eqs.(31) ~ (32) are automatically satisfied. After integration with respect to x in $[0, x]$, Eqs.(29) ~ (30) reduce to

$$\frac{2\mu_{12}^{(1)}}{\pi} \sum_{n=0}^{\infty} \int_0^{\infty} \frac{1}{s} [d_1 a_n G_n^{(1)} \mathrm{J}_{2n+2}(sl) + d_2 b_n G_n^{(2)} \mathrm{J}_{2n+1}(sl)] \sin(sx) \mathrm{d}s = -\sigma_0 x \quad (0 \leqslant x \leqslant l), \tag{39}$$

$$\sum_{n=0}^{\infty} \int_0^{\infty} \frac{1}{s} [d_3 a_n G_n^{(1)} \mathrm{J}_{2n+2}(sl) + d_4 b_n G_n^{(2)} \mathrm{J}_{2n+2}(sl)] [\cos(sx) - 1] \mathrm{d}s = 0 \quad (0 \leqslant x \leqslant l). \tag{40}$$

From the relationships[23]

$$\int_0^{\infty} \frac{1}{s} \mathrm{J}_n(sa) \sin(bs) \mathrm{d}s = \begin{cases} \dfrac{\sin[n \arcsin(b/a)]}{n} & (a > b), \\ \dfrac{a^n \sin(n\pi/2)}{n[b + \sqrt{b^2 - a^2}]^n} & (b > a), \end{cases} \tag{41}$$

$$\int_0^{\infty} \frac{1}{s} \mathrm{J}_n(sa) \cos(bs) \mathrm{d}s = \begin{cases} \dfrac{\cos[n \arcsin(b/a)]}{n} & (a > b), \\ \dfrac{a^n \cos(n\pi/2)}{n[b + \sqrt{b^2 - a^2}]^n} & (b > a), \end{cases} \tag{42}$$

the semi-infinite integral in Eqs.(39) ~ (40) can be evaluated directly. Equations (39) ~ (40) can now be solved for the coefficients a_n and b_n by the Schmidt method[21 ~ 23]. For brevity, Eqs. (39) ~ (40) can be rewritten as

$$\sum_{n=0}^{\infty} a_n E_n^*(x) + \sum_{n=0}^{\infty} b_n F_n^*(x) = U_0(x) \qquad (0 \leqslant x \leqslant l), \tag{43}$$

$$\sum_{n=0}^{\infty} a_n G_n^*(x) + \sum_{n=0}^{\infty} b_n H_n^*(x) = 0 \qquad (0 \leqslant x \leqslant l), \tag{44}$$

where $E_n^*(x)$, $F_n^*(x)$, $G_n^*(x)$ and $H_n^*(x)$ and $U_0(x)$ are known functions. a_n and b_n are unknown coefficients.

From Eq.(44), it can be obtained

$$\sum_{n=0}^{\infty} b_n H_n^*(x) = -\sum_{n=0}^{\infty} a_n G_n^*(x). \tag{45}$$

It can now be solved for the coefficients b_n by the Schmidt method[21 ~ 22,25 ~ 31]. Here the form $-\sum_{n=0}^{\infty} a_n G_n^*(x)$ can be considered as a known function temporarily. A set of functions $P_n(x)$, which satisfy the orthogonality condition

$$\int_0^l P_m(x) P_n(x) \mathrm{d}x = N_n \delta_{mn}, \quad N_n = \int_0^l P_n^2(x) \mathrm{d}x \tag{46}$$

can be constructed from the function, $H_n^*(x)$, such that

$$P_n(x) = \sum_{i=0}^{n} \frac{M_{in}}{M_{nn}} H_i^*(x), \tag{47}$$

where M_{ij} is the cofactor of the element d_{ij} of $\boldsymbol{D}_n$, which is defined as

$$\boldsymbol{D}_n = \begin{bmatrix} d_{00}, d_{01}, d_{02}, \cdots, d_{0n} \\ d_{10}, d_{11}, d_{12}, \cdots, d_{1n} \\ d_{20}, d_{21}, d_{22}, \cdots, d_{2n} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ d_{n0}, d_{n1}, d_{n2}, \cdots, d_{nn} \end{bmatrix}, \quad d_{ij} = \int_0^l H_i^*(x) H_j^*(x) \mathrm{d}x. \tag{48}$$

Using Eqs.(45) ~ (48), we obtain

$$b_n = \sum_{j=n}^{\infty} q_j \frac{M_{nj}}{M_{jj}} \text{ with } q_j = -\sum_{i=0}^{\infty} a_i \frac{1}{N_j} \int_0^l G_i^*(x) P_j(x) \mathrm{d}x. \tag{49}$$

Hence, it can be rewritten

$$b_n = \sum_{i=0}^{\infty} a_i K_{in}^*, \quad K_{in}^* = -\sum_{j=n}^{\infty} \frac{M_{nj}}{N_j M_{jj}} \int_0^l G_i^*(x) P_j(x) \mathrm{d}x. \tag{50}$$

Substituting Eq.(50) into Eq.(43), it can be obtained

$$\sum_{n=0}^{\infty} a_n Y_n^*(x) = U_0(x), \quad Y_n^*(x) = E_n^*(x) + \sum_{i=0}^{\infty} K_{ni}^* F_i^*(x). \tag{51}$$

So it can now be solved for the coefficients a_n by the Schmidt method again as mentioned above. With the aid of Eq.(50), the coefficients b_n can be obtained. When $(E_i^{(1)}, \mu_{ik}^{(1)}, \nu_{ik}^{(1)}) = (E_i^{(2)}, \mu_{ik}^{(2)}, \nu_{ik}^{(2)})$ $(i, k = 1,2,3)$, it can be obtained that $a_n = 0$ $(n = 0,1,2,3,\cdots)$, $b_0 = -(\sigma_0 \sqrt{\pi} l / d_2 \mu_{12}^{(1)})$ and $b_n = 0$ $(n = 1,2,3,4,\cdots)$.

4 Stress Intensity Factors

The coefficients a_n and b_n are known, so that the entire stress field can be obtained. However, in fracture mechanics, it is important in fracture mechanics to determine the stresses $\sigma_{yy}^{(1)}$ and $\sigma_{xy}^{(1)}$ in the vicinity of the crack tips. In the case of the present study, $\sigma_{yy}^{(1)}$ and $\sigma_{xy}^{(1)}$ along the crack line can be expressed as

$$\sigma_{yy}^{(1)}(x,0) = \frac{2\mu_{12}^{(1)}}{\pi} \sum_{n=0}^{\infty} \int_0^{\infty} [d_1 a_n G_n^{(1)} \mathrm{J}_{2n+2}(sl) + d_2 b_n G_n^{(2)} \mathrm{J}_{2n+1}(sl)] \cos(sx) \mathrm{d}s, \tag{52}$$

$$\sigma_{xy}^{(1)}(x,0) = -\frac{2\mu_{12}^{(1)}}{\pi} \sum_{n=0}^{\infty} \int_0^{\infty} [d_3 a_n G_n^{(1)} \mathrm{J}_{2n+2}(sl) + d_4 b_n G_n^{(2)} \mathrm{J}_{2n+1}(sl)] \sin(sx) \mathrm{d}s. \tag{53}$$

When $(E_i^{(1)}, \mu_{ik}^{(1)}, \nu_{ik}^{(1)}) = (E_i^{(2)}, \mu_{ik}^{(2)}, \nu_{ik}^{(2)})$ $(i, k = 1,2,3)$, $\sigma_{yy}^{(1)}$ and $\sigma_{xy}^{(1)}$ along the crack line can be exactly expressed as

$$\sigma_{yy}^{(1)}(x,0) = \frac{2\mu_{12}^{(1)}}{\pi} \int_0^{\infty} d_2 b_0 G_0^{(2)} \mathrm{J}_1(sl) \cos(sx) \mathrm{d}s =$$

$$-\sigma_0 l \int_0^{\infty} \mathrm{J}_1(sl) \cos(sx) \mathrm{d}s = \begin{cases} -\sigma_0 & (x < l), \\ \dfrac{\sigma_0 l^2}{\sqrt{x^2 - l^2}[x + \sqrt{x^2 - l^2}]} & (x > l), \end{cases} \tag{54}$$

$$\sigma_{xy}^{(1)}(x,0) = 0. \tag{55}$$

When the materials of the two half planes are the same, an exact solution can be obtained by use of the Schmidt method. This is also proved that the Schmidt method is performed satisfactorily.

An examination of Eqs. (52) ~ (55) shows that, the singular part of the stress field can be obtained from[23] the relationships as follows:

$$\int_0^\infty J_n(sa)\cos(bs)ds = \begin{cases} \dfrac{\cos[n\arcsin(b/a)]}{\sqrt{a^2-b^2}} & (a > b), \\ -\dfrac{a^n\sin(n\pi/2)}{\sqrt{b^2-a^2}[b+\sqrt{b^2-a^2}]^n} & (b > a), \end{cases}$$

$$\int_0^\infty J_n(sa)\sin(bs)ds = \begin{cases} \dfrac{\sin[n\arcsin(b/a)]}{\sqrt{a^2-b^2}} & (a > b), \\ \dfrac{a^n\cos(n\pi/2)}{\sqrt{b^2-a^2}[b+\sqrt{b^2-a^2}]^n} & (b > a). \end{cases}$$

The singular part of the stress field can be expressed respectively as follows ($l < x$):

$$\sigma(x) = \frac{2d_2\mu_{12}^{(1)}}{\pi}\sum_{n=0}^\infty b_n G_n^{(2)} H_n^{(1)}(x), \tag{56}$$

$$\tau(x) = -\frac{2d_3\mu_{12}^{(1)}}{\pi}\sum_{n=0}^\infty a_n G_n^{(1)} H_n^{(2)}(x), \tag{57}$$

where $H_n^{(1)}(x) = \dfrac{(-1)^{n+1} l^{2n+1}}{\sqrt{x^2-l^2}[x+\sqrt{x^2-l^2}]^{2n+1}}$, $H_n^{(2)}(x) = \dfrac{(-1)^{n+1} l^{2n+2}}{\sqrt{x^2-l^2}[x+\sqrt{x^2-l^2}]^{2n+2}}$.

The stress intensity factors K_{I} and K_{II} can be written as follows:

$$K_{\mathrm{I}} = \lim_{x\to l^+}\sqrt{2\pi(x-l)}\,\sigma(x) = \frac{2d_2\mu_{12}^{(1)}}{\sqrt{l}}\sum_{n=0}^\infty b_n\frac{\Gamma(2n+1+1/2)}{(2n)!}, \tag{58}$$

$$K_{\mathrm{II}} = \lim_{x\to l^+}\sqrt{2\pi(x-l)}\,\tau(x) = -\frac{2d_3\mu_{12}^{(1)}}{\sqrt{l}}\sum_{n=0}^\infty a_n\frac{\Gamma(2n+2+1/2)}{(2n+1)!}. \tag{59}$$

When $(E_i^{(1)}, \mu_{ik}^{(1)}, \nu_{ik}^{(1)}) = (E_i^{(2)}, \mu_{ik}^{(2)}, \nu_{ik}^{(2)})$ $(i, k = 1,2,3)$, the stress intensity factors K_{I} and K_{II} can be exactly written as follows:

$$K_{\mathrm{I}} = \sigma_0\sqrt{\pi l},\ K_{\mathrm{II}} = 0. \tag{60}$$

5 Numerical Calculations and Discussion

As discussed in Refs. [21,22,25 ~ 31], it can be seen that the Schmidt method is performed satisfactorily if the first ten terms of infinite series to Eqs. (43) ~ (44) are retained. At $-l \leqslant x \leqslant l$, $y = 0$, it can be obtained that $\sigma_y^{(1)}/\sigma_0$ is very close to negative unity. Hence, the solution of the present paper can also be proved to satisfy the boundary conditions (8). The dimensionless stress intensity factors K/σ_0 are calculated numerically. As an example, the numerical results of the present paper are shown in Fig. 2. From the results, the following observations are very significant:

(i) Contrary to the previous solution of the interface crack, it is found that the stress singularity of the present interface crack solution is of the same nature as that for the ordinary crack in homogeneous materials.

(ii) When the materials of the two half planes are the same, an exact solution can be obtained by use of the Schmidt method. This is also proved that the Schmidt method is performed satisfactorily.

(iii) From the results, the stress intensity factors are independent of material constants. This is the same as the conclusion in Ref.[9]. However, in Refs.[1 ~ 3,7], the results of the stress intensity factors are dependent of materials from the two half planes.

(iv) In the present paper, the unknown variables of dual integral equations are the *displacement across the crack surfaces. However, in the previous works, the* unknown variables of dual integral equations are the dislocation density functions. This is the major difference.

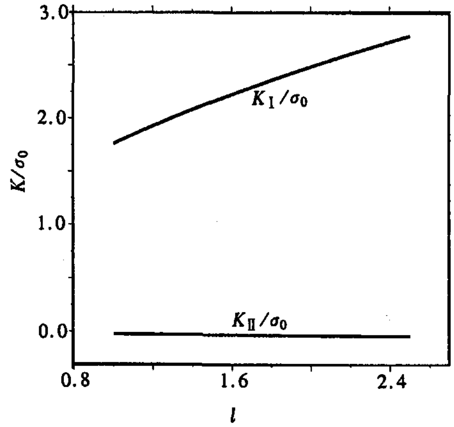

Fig.2 The stress intensity factor versus l

(v) In this paper, we give a new approach to solve the opening interface crack problem. During the solving process, the mathematical difficulties are not met, i.e. the oscillatory stress singularity and the overlapping of the crack surfaces do not meet.

(vi) The stress intensity factors K_{I}/σ_0 increase almost linearly when the length of the crack increases as shown in Fig.2.

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Appendix

$$H_1 = c_{12}^{(1)} - c_{22}^{(1)}\alpha_1\gamma_1,\ H_2 = c_{12}^{(1)} - c_{22}^{(1)}\alpha_2\gamma_2,\ H_3 = c_{12}^{(1)} - c_{22}^{(1)}\alpha_3\gamma_3,\ H_4 = c_{12}^{(1)} - c_{22}^{(1)}\alpha_4\gamma_4,$$
$$F_1 = \alpha_1 + \gamma_1,\ F_2 = \alpha_2 + \gamma_2,\ F_3 = \alpha_3 + \gamma_3,\ F_4 = \alpha_4 + \gamma_4,$$
$$P_1 = \alpha_2\mu_{12}^{(2)}(H_4F_3\mu_{12}^{(2)} - H_3F_4\mu_{12}^{(2)} - H_3F_1\mu_{12}^{(1)} + H_4F_1\mu_{12}^{(1)} - H_1F_3\mu_{12}^{(1)} + H_1F_4\mu_{12}^{(1)}),$$
$$P_2 = \alpha_1\mu_{12}^{(2)}[H_3F_4\mu_{12}^{(2)} + H_3F_2\mu_{12}^{(1)} + H_2F_3\mu_{12}^{(1)} - H_2F_4\mu_{12}^{(1)} - H_4(F_3\mu_{12}^{(2)} + F_2\mu_{12}^{(1)})],$$
$$P_3 = \alpha_4\mu_{12}^{(1)}[H_3(-F_1 + F_2)\mu_{12}^{(2)} - H_1F_3\mu_{12}^{(2)} + H_2F_3\mu_{12}^{(2)} + H_2F_1\mu_{12}^{(1)} - H_1F_2\mu_{12}^{(1)}],$$
$$P_4 = \alpha_3\mu_{12}^{(1)}[H_4(F_1 - F_2)\mu_{12}^{(2)} + H_1F_4\mu_{12}^{(2)} - H_2F_4\mu_{12}^{(2)} - H_2F_1\mu_{12}^{(1)} + H_1F_2\mu_{12}^{(1)}],$$
$$P_0 = P_1 + P_2 + P_3 + P_4,$$
$$R_1 = \mu_{12}^{(2)^2}[\alpha_2H_1(H_4F_3 - H_3F_4) + \alpha_1H_2(H_3F_4 - H_4F_3)],$$
$$R_2 = \mu_{12}^{(1)}\mu_{12}^{(2)}[\alpha_3H_2H_4F_1 + \alpha_4H_1H_3F_2 - \alpha_3H_1H_4F_2 - \alpha_4H_2H_3F_1),$$
$$R_3 = \mu_{12}^{(2)^2}(-H_1H_4F_3 + H_2H_4F_3 + H_1H_3F_4 - H_2H_3F_4),$$
$$R_4 = \mu_{12}^{(1)}\mu_{12}^{(2)}(-H_2H_3F_1 + H_2H_4F_1 + H_1H_3F_2 - H_1H_4F_2),$$
$$Q_1 = \mu_{12}^{(2)^2}[-H_4\alpha_1F_2F_3 + \alpha_1H_3F_2F_4 + \alpha_2F_1(H_4F_3 - H_3F_4),$$
$$Q_2 = \mu_{12}^{(1)}\mu_{12}^{(2)}(-\alpha_4H_1F_2F_3 - \alpha_3H_2F_1F_4 + \alpha_3H_1F_2F_4 + \alpha_4H_2F_1F_3),$$
$$Q_3 = \mu_{12}^{(2)^2}[-H_4(F_1 - F_2)F_3 + H_3F_1F_4 - H_3F_2F_4],$$
$$Q_4 = \mu_{12}^{(1)}\mu_{12}^{(2)}(-H_1F_2F_3 - H_2F_1F_4 + H_1F_2F_4 + H_2F_1F_3),$$
$$d_1 = (R_1 + R_2)/P_0,\ d_2 = (R_3 + R_4)/P_0,\ d_3 = (Q_1 + Q_2)/P_0,\ d_4 = (Q_3 + Q_4)/P_0.$$